Introduction	Background 0000	The example ০০০০০০০	Results oo	Conclusions

Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

University of São Paulo, Brasil; Durham Univeristy, UK

WPMSIIP - September 2009

Solving act-state independent imprecise decision processes

Introduction	Background	The example	Results	Conclusions



- Authors
- Objectives



- 3 The example
 - Problem formulation
 - Implementation

4 Results



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Introduction	Background	The example	Results 00	Conclusions

From the University of São Paulo

Ricardo Shirota Filho

- Ph.D. student at the University of São Paulo
- Supervised by Prof. Fabio G. Cozman
- Currently visiting Durham University
 - Markov Decision Processes (sequential decision making)
 - Artificial Intelligence Planning
 - Imprecise probabilities
 - Algorithms

Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction ○●○○	Background 0000	The example	Results oo	Conclusions
From Du	rham Univor	sity		

From Durham University

Matthias C. M. Troffaes

- Lecturer at Durham University
 - Foundations of statistics
 - Sequential decision making
 - Imprecise probabilities
 - Reliability, fault trees

Nathan Huntley

- Ph.D. student at Durham University
- Supervised by Matthias Troffaes
 - Sequential decision making
 - Imprecise probabilities

Introduction	Background	The example	Results oo	Conclusions

After a few discussions...

A common topic of interest

Investigate conditions for optimality in sequential decision making with imprecision under fairly general assumptions

- Ricardo: Markov Decision Processes, applications
- Matthias and Nathan: decision trees, arbitrary choice functions, gambles

Solving act-state independent imprecise decision processes

Introduction	Background	The example	Results oo	Conclusions
The menu				

In this presentation

- General description and current results
- A simple illustrative example

Later...

Matthias: Locality property and implications for foundations

Nathan: Implications for backward induction

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Solving act-state independent imprecise decision processes

Introduction	Background	The example	Results oo	Conclusions
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Sequential decision making





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Sequential decision making





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Introduction	Background o●oo	The example	Results oo	Conclusions
Assumpt	ions			

- No probabilities are assumed and rewards do not need to be expressed in terms of utility, instead we use arbitrary choice functions
- Rewards can depend on full state history
- State and action spaces can depend on the stage

However, not everything is perfect...

imiting condition

• Act-state independence could not be avoided

Solving act-state independent imprecise decision processes

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background ooeo	The example	Results oo	Conclusions 00
Our main	n result			

Locality property

$$X + \bigoplus_{s_k} E_{s_k} Y(s_k) \in \operatorname{opt} \left(\mathcal{X} + \bigoplus_{s_k} E_{s_k} \mathcal{Y}(s_k) \middle| h_{k-1} \right) \iff$$
$$X \in \operatorname{opt}(\mathcal{X}|h_{k-1}) \text{ and } Y(s_k) \in \operatorname{opt}(\mathcal{Y}(s_k)|h_{k-1}s_k) \text{ for all } s_k.$$

It can be shown that

The locality property is necessary and sufficient for normal form solutions to reduce to a sequence of single stage normal form solutions.

Solving act-state independent imprecise decision processes

Introduction	Background ooeo	The example	Results oo	Conclusions 00
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Solving act-state independent imprecise decision processes

Introduction	Background ○○○●	The example	Results oo	Conclusions
Additiona	al results			

When considering:

- Lower previsions
- Rewards expressed in terms of utility

Maximality and E-admissibility

- Strictly positive lower probabilities
- Marginal extension

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Introduction	Background ○○○●	The example	Results oo	Conclusions
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Solving act-state independent imprecise decision processes

Introduction	Background	The example ●oooooo	Results oo	Conclusions 00
The simp	le coin probl	em		

An agent bets sequentially on a coin.

He recieves one utile on correct prediction and loses one otherwise.

The agent's objective is to perform optimally, thus maximizing the expected profit over the sequence of coin tosses.

Solving act-state independent imprecise decision processes Ricardo S

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Solving act-state independent imprecise decision processes

Introduction	Background	The example	Results	Conclusions		
	0000	●000000	oo	00		
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The simp	ole coin probl	em		

- The bias of the coin is not known (Bayesian agent is no longer an obvious choice)
- The toss is not affected by the decision (act-state independence)

And to make things more interesting...

• Learning is considered (full state history is available)

Solving act-state independent imprecise decision processes

Introduction	Background	The example	Results	Conclusions
0000		o●ooooo	oo	00
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Solving act-state independent imprecise decision processes F

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	0000	o●ooooo	oo	00
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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example	Results	Conclusions
0000		o●ooooo	oo	00
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Introduction	Background 0000	The example oo●oooo	Results oo	Conclusions
Formulation	on			

States (possible outcomes)

- heads
- tails
- Decisions (possible bets)
 - heads
 - tails
- Rewards (expressed in utility)
 - if bet = outcome, receive 1
 - if bet \neq outcome, pay 1
- Transition probabilities (IDM)
 - Vacuous prior
 - Updated using observed transition

Solving act-state independent imprecise decision processes

Introduction	Background	The example oo●oooo	Results oo	Conclusions
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Correculat	lon			
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Introduction	Background	The example	Results	Conclusions

Formulation

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Introduction	Background	The example oo●oooo	Results oo	Conclusions
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Introduction	Background	The example ○○○●○○○	Results oo	Conclusions 00
Formulation				

Notice that this example is similar to an MDPIP, because

- We assume that our uncertainty is expressed by a credal set
- Our rewards are expressed in terms of utilities

However, it is not an MDPIP because

• The Markov assumption does not hold (probabilities depend on the full state history)

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example ○○○●○○○	Results oo	Conclusions 00
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Introduction	Background	The example oooo●oo	Results oo	Conclusions

The Imprecise Dirichlet Model

Predictive lower prevision

$$\underline{E}(X|h_k) = \sum_{i} \left(\frac{n_i}{N+s} X(i) \right) + \frac{s}{N+s} \inf_{t \in \Delta} \left(\sum_{i} t_i X(i) \right)$$

Optimality criteria:

- Γ -maximin ($\underline{E}(X) > \underline{E}(Y)$)
- Interval dominance $(\underline{E}(X) > \overline{E}(Y))$
- Maximality $(\underline{E}(X Y) > 0)$
- E-admissibility

Solving act-state independent imprecise decision processes

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Introduction Backgro	und The example	Results oo	Conclusions

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Implementation				
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Introduction	Background	The example	Results	Conclusions

Implementation

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●○	Results oo	Conclusions
Impleme	ntation			

Python

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●○	Results oo	Conclusions
Impleme	ntation			

- Python
- Bayesian agent
 - Precise probabilities (distribution)
 - Uniform prior
- IDM agent
 - Vacuous prior
 - Maximality
 - E-admissibility

Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●○	Results oo	Conclusions
Impleme	ntation			

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Impleme	ntation			

- 10 coin tosses
- 10,000 experiments
- Average gain

In particular

- Bayesian = Γ-maximin
- Interval dominance = maximality = E-admissibility

Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Impleme	ntation			

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Impleme	ntation			

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Solving act-state independent imprecise decision processes Rica

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Implemer	ntation			

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Implemer	ntation			

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Solving act-state independent imprecise decision processes

Introduction	Background	The example ○○○○○●	Results oo	Conclusions
Implemer	ntation			

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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I a second a	Introduction 0000	Background	The example	Results ●○	Conclusions





Solving act-state independent imprecise decision processes

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Introduction	Background	The example	Results ○●	Conclusions 00
D				

Bayesian vs Maximal agent with no-bet option



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Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background 0000	The example	Results oo	Conclusions ●○			
Conclusi	Conclusions						

- For best performance, forget about imprecise probabilities and simply be a happy Bayesian
- Por robustness, avoid Γ-maximin (or Bayesian agent) and instead adopt maximality or E-admissibility

Solving act-state independent imprecise decision processes

Introduction	Background 0000	The example	Results oo	Conclusions ●○
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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example	Results oo	Conclusions ○●
Future st	eps			

- Γ-maximin and interval dominance?
- More complex problems (e.g. Peter Walley's bag of marbles)...
- B Real applications?

Ultimate goal

Act-state dependence

Questions? Comments?

Thank you for your attention!

Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background 0000	The example	Results oo	Conclusions ○●
Future ste	eps			

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example	Results oo	Conclusions ○●
Future st	eps			

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background 0000	The example	Results oo	Conclusions ○●
Future st	eps			

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example	Results oo	Conclusions ○●
Future st	eps			

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Solving act-state independent imprecise decision processes

Ricardo Shirota Filho, Matthias C. M. Troffaes, Nathan Huntley

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Introduction	Background	The example	Results oo	Conclusions ○●
Future st	eps			

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Solving act-state independent imprecise decision processes

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Introduction	Background	The example	Results oo	Conclusions ○●
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