Nonparametric Predictive Inference for System Reliability

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WPMSIIP'09, Munich, September 2009

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NPI for a k-out-of-m system

Data: s successes from n components tested, these are exchangeable with components in system

NPI lower and upper probabilities for k-out-of-m to function, with $0 < s < n$:

$$\overline{P}(m : k|\, n, s) = \binom{n+m}{n}^{-1} \times \left[\binom{s+k}{s}\binom{n-s+m-k}{n-s} + \sum_{l=k+1}^{m} \binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s} \right]$$

$$\underline{P}(m : k|\, n, s) = 1 - \binom{n+m}{n}^{-1} \left[\sum_{l=0}^{k-1} \binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s} \right]$$

Special cases:

$$[\underline{P}, \overline{P}](1 : 1|\, n, s) = [\frac{s}{n+1}, \frac{s+1}{n+1}]$$

$$[\underline{P}, \overline{P}](m : k|\, n, n) = [1 - \binom{n+k-1}{n}\binom{n+m}{n}^{-1}, 1]$$

$$[\underline{P}, \overline{P}](m : k|\, n, 0) = [0, \binom{n+m-k}{n}\binom{n+m}{n}^{-1}]$$

n	s	$k=58$ $\underline{P}$	$\overline{P}$	$k=59$ $\underline{P}$	$\overline{P}$	$k=60$ $\underline{P}$	$\overline{P}$	$k=61$ $\underline{P}$	$\overline{P}$	$k=62$ $\underline{P}$	$\overline{P}$
1	1	0.079	1	0.063	1	0.048	1	0.032	1	0.016	1
2	2	0.151	1	0.122	1	0.092	1	0.062	1	0.031	1
3	3	0.217	1	0.176	1	0.134	1	0.091	1	0.046	1
	2	0.021	0.217	0.014	0.176	0.008	0.134	0.004	0.091	0.001	0.046
5	5	0.330	1	0.272	1	0.211	1	0.145	1	0.075	1
	4	0.060	0.330	0.041	0.272	0.025	0.211	0.013	0.145	0.005	0.075
10	10	0.538	1	0.458	1	0.367	1	0.260	1	0.139	1
	9	0.192	0.538	0.139	0.458	0.090	0.367	0.049	0.260	0.018	0.139
	8	0.051	0.192	0.032	0.139	0.017	0.090	0.007	0.049	0.002	0.018
	7	0.011	0.051	0.006	0.032	0.003	0.017	0.001	0.007	0.000	0.002
20	20	0.763	1	0.681	1	0.573	1	0.431	1	0.244	1
30	30	0.868	1	0.800	1	0.699	1	0.548	1	0.326	1
40	40	0.922	1	0.867	1	0.780	1	0.633	1	0.392	1
50	50	0.952	1	0.910	1	0.834	1	0.696	1	0.446	1
60	60	0.969	1	0.936	1	0.872	1	0.744	1	0.492	1
62	62	0.971	1	0.941	1	0.878	1	0.752	1	0.500	1
100	100	0.993	1	0.980	1	0.946	1	0.855	1	0.617	1

Table 1: NPI lower and upper probabilities for a k-out-of-62 system

Series system containing two k_i-out-of-m_i subsystems, one type of component:

$$\underline{P}(m_1 : k_1, m_2 : k_2 \mid n, s) = \binom{n + m_1 + m_2}{n, m_1, m_2}^{-1} \times \sum_{l_1=k_1}^{m_1} \sum_{l_2=k_2}^{m_2} \binom{s - 1 + l_1 + l_2}{s - 1, l_1, l_2} \binom{n - s + m_1 - l_1 + m_2 - l_2}{n - s, m_1 - l_1, m_2 - l_2}$$

$$\begin{aligned}\overline{P}(m_1 : k_1, m_2 : k_2 \mid n, s) = \binom{n + m_1 + m_2}{n, m_1, m_2}^{-1} \times \Bigg[\quad & \sum_{l_1=k_1}^{m_1} \binom{s + l_1 + k_2 - 1}{s, l_1, k_2 - 1} \binom{n - s + m_1 - l_1 + m_2 - k_2}{n - s, m_1 - l_1, m_2 - k_2} + \\ & \sum_{l_2=k_2}^{m_2} \binom{s + k_1 - 1 + l_2}{s, k_1 - 1, l_2} \binom{n - s + m_1 - k_1 + m_2 - l_2}{n - s, m_1 - k_1, m_2 - l_2} + \\ & \sum_{l_1=k_1}^{m_1} \sum_{l_2=k_2}^{m_2} \binom{s - 1 + l_1 + l_2}{s - 1, l_1, l_2} \binom{n - s + m_1 - l_1 + m_2 - l_2}{n - s, m_1 - l_1, m_2 - l_2} \Bigg]\end{aligned}$$

		$k_1 = k_2 = 29$		$k_1 = 29, k_2 = 30$		$k_1 = k_2 = 30$		$k_1 = k_2 = 31$	
n	s	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$
1	1	0.066	1	0.050	1	0.040	1	0.016	1
2	2	0.126	1	0.096	1	0.077	1	0.031	1
3	3	0.182	1	0.139	1	0.113	1	0.046	1
	2	0.015	0.182	0.010	0.139	0.006	0.113	0.001	0.046
5	5	0.280	1	0.218	1	0.178	1	0.075	1
	4	0.045	0.280	0.028	0.218	0.019	0.178	0.005	0.075
10	10	0.467	1	0.375	1	0.314	1	0.139	1
	9	0.148	0.467	0.099	0.375	0.070	0.314	0.018	0.139
	8	0.036	0.148	0.020	0.099	0.012	0.070	0.002	0.018
	7	0.007	0.036	0.003	0.020	0.002	0.012	0.000	0.002
20	20	0.687	1	0.579	1	0.503	1	0.244	1
30	30	0.803	1	0.701	1	0.625	1	0.326	1
40	40	0.868	1	0.778	1	0.708	1	0.392	1
50	50	0.908	1	0.829	1	0.766	1	0.446	1
60	60	0.934	1	0.865	1	0.809	1	0.492	1
62	62	0.938	1	0.871	1	0.816	1	0.500	1
100	100	0.977	1	0.936	1	0.901	1	0.617	1

Table 2: NPI lower and upper probabilities with $m_1 = m_2 = 31$

Series system containing three k_i-out-of-m_i subsystems, one type of component:

$$\underline{P}(m_1 : k_1, m_2 : k_2, m_3 : k_3 \mid n, s) = \binom{n + m_1 + m_2 + m_3}{n, m_1, m_2, m_3}^{-1} \times \sum_{l_1=k_1}^{m_1} \sum_{l_2=k_2}^{m_2} \sum_{l_3=k_3}^{m_3} \binom{s - 1 + l_1 + l_2 + l_3}{s - 1, l_1, l_2, l_3} \binom{n - s + m_1 - l_1 + m_2 - l_2 + m_3 - l_3}{n - s, m_1 - l_1, m_2 - l_2, m_3 - l_3}$$

$$\begin{aligned}
&\overline{P}(m_1 : k_1, m_2 : k_2, m_3 : k_3 \mid n, s) = \\
&\quad \binom{n + m_1 + m_2 + m_3}{n, m_1, m_2, m_3}^{-1} \times \Bigg[\sum_{l_2=k_2}^{m_2} \sum_{l_3=k_3}^{m_3} \binom{s + k_1 - 1 + l_2 + l_3}{s, k_1 - 1, l_2, l_3} \binom{n - s + m_1 - k_1 + m_2 - l_2 + m_3 - l_3}{n - s, m_1 - k_1, m_2 - l_2, m_3 - l_3} + \\
&\quad \sum_{l_1=k_1}^{m_1} \sum_{l_3=k_3}^{m_3} \binom{s + l_1 + k_2 - 1 + l_3}{s, l_1, k_2 - 1, l_3} \binom{n - s + m_1 - l_1 + m_2 - k_2 + m_3 - l_3}{n - s, m_1 - l_1, m_2 - k_2, m_3 - l_3} + \\
&\quad \sum_{l_1=k_1}^{m_1} \sum_{l_2=k_2}^{m_2} \binom{s + l_1 + l_2 + k_3 - 1}{s, l_1, l_2, k_3 - 1} \binom{n - s + m_1 - l_1 + m_2 - l_2 + m_3 - k_3}{n - s, m_1 - l_1, m_2 - l_2, m_3 - k_3} + \\
&\quad \sum_{l_1=k_1}^{m_1} \sum_{l_2=k_2}^{m_2} \sum_{l_3=k_3}^{m_3} \binom{s - 1 + l_1 + l_2 + l_3}{s - 1, l_1, l_2, l_3} \binom{n - s + m_1 - l_1 + m_2 - l_2 + m_3 - l_3}{n - s, m_1 - l_1, m_2 - l_2, m_3 - l_3} \Bigg]
\end{aligned}$$

		$k=15, m=15$		$k=16, m=16$		$k=30, m=31$		$k=31, m=31$	
n	s	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$
1	1	0.063	1	0.059	1	0.063	1	0.031	1
2	2	0.118	1	0.111	1	0.119	1	0.061	1
3	3	0.167	1	0.158	1	0.171	1	0.088	1
	2	0.020	0.167	0.018	0.158	0.016	0.171	0.005	0.088
5	5	0.250	1	0.238	1	0.262	1	0.139	1
	4	0.053	0.250	0.048	0.238	0.045	0.262	0.016	0.139
10	10	0.400	1	0.385	1	0.433	1	0.244	1
	9	0.150	0.400	0.138	0.385	0.142	0.433	0.055	0.244
	8	0.052	0.15	0.046	0.138	0.039	0.142	0.011	0.055
	7	0.017	0.052	0.014	0.046	0.009	0.039	0.002	0.011
20	20	0.571	1	0.556	1	0.635	1	0.392	1
30	30	0.667	1	0.651	1	0.746	1	0.492	1
40	40	0.727	1	0.714	1	0.813	1	0.563	1
50	50	0.769	1	0.758	1	0.856	1	0.617	1
60	60	0.800	1	0.789	1	0.886	1	0.659	1
62	62	0.805	1	0.795	1	0.891	1	0.667	1
100	100	0.870	1	0.862	1	0.945	1	0.763	1

Table 3: NPI lower and upper probabilities for k-out-of-m systems

(k_1, k_2, k_3) :		(14, 15, 30)		(15, 16, 30)		(15, 16, 31)	
n	s	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$
1	1	0.045	1	0.024	1	0.016	1
2	2	0.087	1	0.047	1	0.031	1
3	3	0.127	1	0.069	1	0.046	1
	2	0.008	0.127	0.003	0.069	0.001	0.046
5	5	0.197	1	0.110	1	0.075	1
	4	0.024	0.197	0.009	0.110	0.005	0.075
10	10	0.345	1	0.200	1	0.139	1
	9	0.085	0.345	0.033	0.200	0.018	0.139
	8	0.016	0.085	0.005	0.033	0.002	0.018
	7	0.003	0.016	0.001	0.005	0.000	0.002
20	20	0.542	1	0.337	1	0.244	1
30	30	0.664	1	0.437	1	0.326	1
40	40	0.744	1	0.513	1	0.392	1
50	50	0.799	1	0.571	1	0.446	1
60	60	0.838	1	0.618	1	0.492	1
62	62	0.844	1	0.626	1	0.500	1
100	100	0.919	1	0.736	1	0.617	1

Table 4: NPI lower and upper probabilities with $m_1 = 15, m_2 = 16, m_3 = 31$

Challenges:

- Redundancy allocation
- k-out-of-m system with different types of component
- More general system structures
- Further applications