Blatt 7 Notizen

Nr. 1

a) $$\frac{f(x+\Delta) - f(x)}{\Delta} = \frac{x+\Delta - x}{\Delta} = \frac{\Delta}{\Delta} = 1$$

b) $$\frac{f(x+\Delta) - f(x)}{\Delta} = \frac{c - c}{\Delta} = \frac{0}{\Delta} = 0$$

c) $$\frac{f(x+\Delta) - f(x)}{\Delta} = \frac{(x+\Delta)^4 - x^4}{\Delta} = \frac{(x+\Delta)^2 (x+\Delta)^2 - x^4}{\Delta}$$

$$= \frac{(x^2 + 2x\Delta + \Delta^2)(x^2 + 2x\Delta + \Delta^2) - x^4}{\Delta}$$

$$= \frac{x^4 + 2\Delta x^3 + x^2\Delta^2 + 2\Delta x^3 + 4\Delta^2 x^2 + 2\Delta^3 x + x^2\Delta^2 + 2\Delta^3 x + \Delta^4 - x^4}{\Delta}$$

$$= \frac{4\Delta x^3 + 2x^2\Delta^2 + 4\Delta^2 x^2 + 4\Delta^3 x + \Delta^4}{\Delta}$$

$$= 4x^3 + 2x^2\Delta + 4x^2\Delta + 4\Delta^2$$

für $\Delta \to 0$ erhält man $f'(x) = 4x^3$.

N.2

a) $f'(x) = nx^{n-1}$

b) $f'(x) = \exp(x) \cdot 1$ (allgemein $\left(\exp(u(x))\right)' = \exp(u(x))\, u'(x)$)

c) $f'(x) = \cos(x)$

d) $f'(x) = \cos^2(x) - \sin^2(x)$

e) $f'(x) = 1 + \tan^2(x)$

f) $f'(x) = -\frac{2x}{(1+x^2)^2}$

g) $f'(x) = \exp(\exp(x))\exp(x)$

h) allgemein: $\left(\ln(u(x))\right)' = \frac{u'(x)}{u(x)}$

hier: $f'(x) = \frac{1}{x} \cdot 1$

i) $f'(x) = 2x$

N.3

a) $f'(x) = 3x^2$; $f''(x) = 6x$

b) $f'(x) = \cos(x)$; $f''(x) = -\sin(x)$

d) $f'(x) = \exp(x^2) \cdot 2x$; $f''(x) = 4x^2 \exp(x^2) + 2\exp(x^2)$

c) $f'(x) = \lambda \exp(\lambda x)$; $f''(x) = \lambda^2 \exp(\lambda x)$

e) $f'(x) = \frac{1-x^2}{(1+x^2)^2}$; $f''(x) = \frac{-2x(1+x^2)^2 - (1-x^2)(4x(1+x^2))}{(1+x^2)^4}$

N.4

a) $f'(x) = \frac{1}{1+x^2}$

b) $f'(x) = 0$

c) $f'(x) = a^x \ln(a)$