Linear regression with NPI

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Second Workshop on Principles and Methods of Statistical Inference with Interval Probability
Munich, September 2009

Outline

Assumptions

- *Nonparametric predictive inference (NPI)* model (Coolen and Coolen-Schrijner (2000), Coolen and Van der Laan (2001), Augustin and Coolen (2004)) is based on Hill's assumption $A_{(n)}$ (Hill (1968)):
- n observations $x_1, \ldots, x_n$ are given corresponding to r.v.s $X_1, \ldots, X_n$, enumerated in the increasing order, $x_0 := -\infty$ and $x_{n+1} = \infty$:

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- Basic assumption: $P(X_{n+1} \in [x_i, x_{i+1}]) = \dfrac{1}{n+1}$ for all $i = 0, \ldots, n$.

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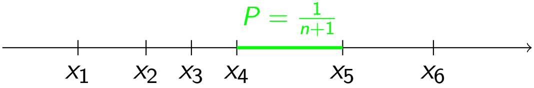

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Lower and upper expectations

- The probability assignments under the basic assumption can be extended to a lower and an upper probability $\underline{P}$ and $\overline{P}$ on the σ-field of all Borel subsets of $\mathbb{R}$.
- For linear regression we need (conditional) expectations.
- In our case we can calculate the lower and the upper expectations.
- Problem: The lower and the upper expectation under NPI are $\pm\infty$ respectively regardless of the values of $x_1, \dots, x_n$.

Bounded NPI

- The only solution is to bound the possible values of X_{n+1}.
- The assumption added: $\underline{P}(X_{n+1} \in [L, U]) = 1$ for some lower and upper bounds L and U respectively:

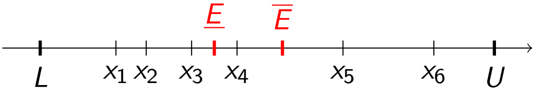
Calculation of the lower and upper expectations

- Let the points $x_1, \dots, x_n$ and $x_0 := L, x_{n+1} := U$ be given.
- Under the above assumptions we have:

$$\underline{E}(X_{n+1} | x_0, \dots, x_{n+1}) = \frac{1}{n+1} \sum_{i=0}^{n} x_i$$

and

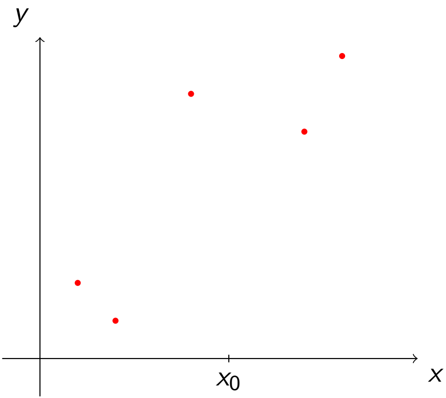
$$\overline{E}(X_{n+1} | x_0, \dots, x_{n+1}) = \frac{1}{n+1} \sum_{i=1}^{n+1} x_i$$



Combining linearity assumption with NPI

- Linear regression is applied when a linear relationship between dependent and independent varible(s) is assumed.
- A regression estimate is then obtained as a conditional expectation calculated on the basis of this assumption, the data and the value(s) of the independent variable(s).
- To apply NPI, a set of real valued points is needed for each possible value (set of values) of independent (variables).

A possible approach



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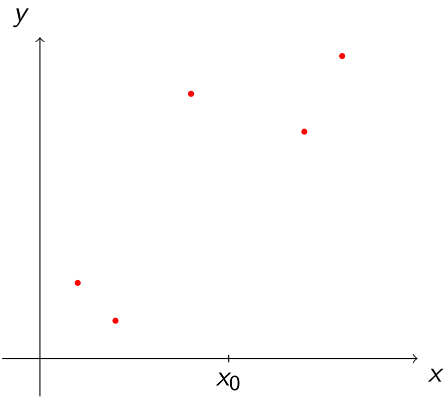

- Connect data points with lines (linearity assumption).

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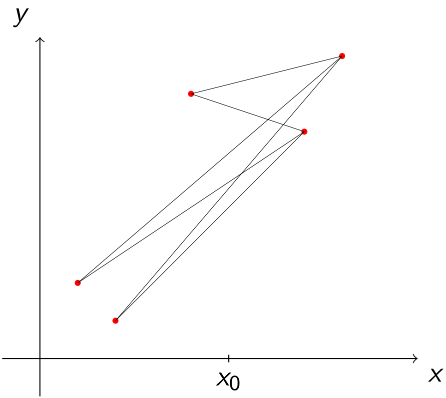

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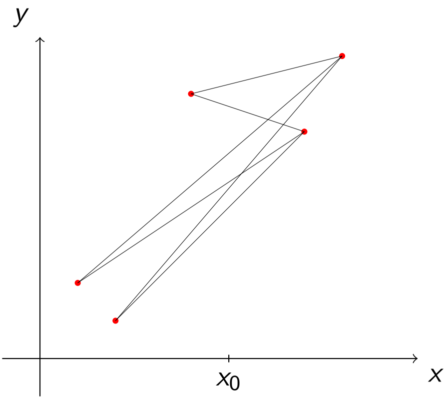

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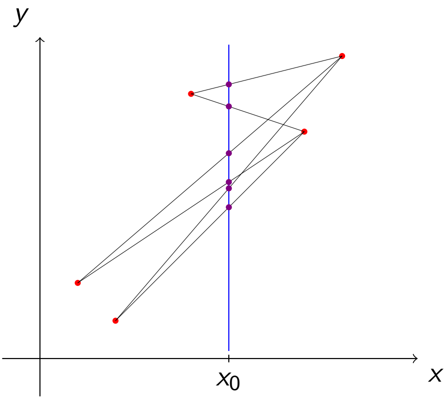

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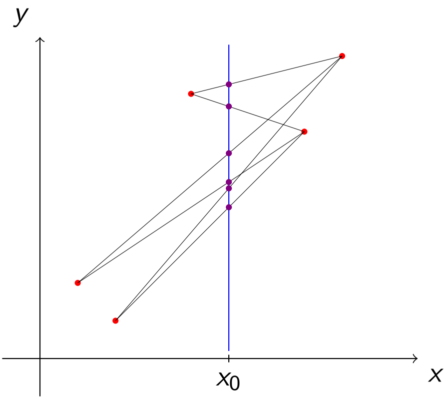

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- Some of the lines intersect the vertical line $x = x_0$.
- The intersection points can be used for NPI estimation.

Example

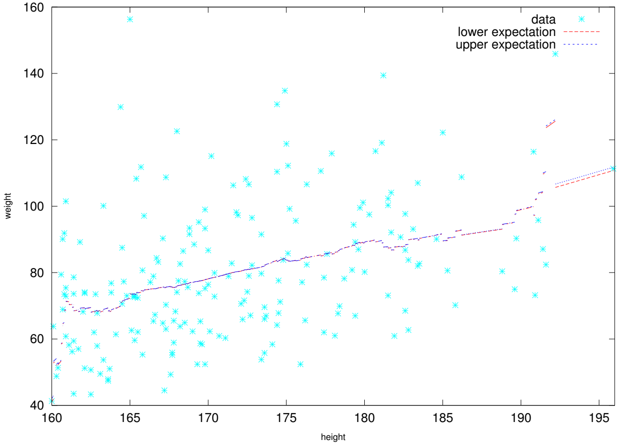

Figure: A demonstrantion of the NPI linear regression on real data: Hable (2009)

Multiple regression

- The method can be generalised to the case with multiple independent variables.
- Take a point $\mathbf{x}_0$.
- Instead of lines, we take n-dimensional hyperplanes that are defined by $n+1$ data points.
- We only consider those hyperplanes where $\mathbf{x}_0$ is a convex combination of the set of independent parts of the data points.
- Data points used for NPI are the intersections between the hyperplanes and the line $\mathbf{x} = \mathbf{x}_0$.